UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Sheet 11

Due date: Jan 17

Problem 1 (The Baumslag–Solitar group $B(1,2)$)**.** The **Baumslag–Solitar group** $B(1,2)$ is the group defined by the presentation $\langle a, b \mid bab^{-1} = a^2 \rangle$.

(a) Show that every element of $B(1,2)$ can be written in the form $b^{-j}a^kb^l$, for $j, l \in \mathbb{Z}_{\geq 0}$ and $k \in \mathbb{Z}$ satisfying the following additional condition: k is odd or $j \cdot l = 0$.

(b) Prove that the exponents in part (a) are uniquely determined by the element in $B(1,2)$.

Hint: Consider the representation $\rho\colon B(1,2) \to \mathrm{GL}_2(\mathbb{Q})$ given by

$$\rho(a) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \rho(b) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

(c) Show that the word problem for $B(1,2)$ is solvable with respect to the presentation $\langle a, b \mid bab^{-1} = a^2 \rangle$.

Problem 2 (Groups of intermediate growth)**.** Let G be an infinite finitely generated group having the following two properties:

(a) G is not of exponential growth;

(b) G is quasi-isometric to $G \times G$.

Prove that G is of intermediate growth.