UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Sheet 11

Due date: July 8

Problem 1 (The standard uniqueness argument)**.** Recall that a **gallery** connecting two chambers C, C' in a chamber complex is a sequence of chambers $C = C_0, \dots, C_n = C'$ where every two consecutive C_i share a panel. The gallery is called **non-stuttering** if no two consecutive C_i are equal.

(a) Let X be a chamber complex where every two chambers are connected by a gallery, let Y be a thin chamber complex of the same dimension, and let $\varphi_1, \varphi_2 \colon X \to Y$ be chamber maps sending non-stuttering galleries of X to non-stuttering galleries of Y. Show that if $\varphi_1|_C = \varphi_2|_C$ for some chamber C of X, then $\varphi_1 = \varphi_2$.

(b) Show that the group of label-preserving automorphisms of the Coxeter complex of type (W, S) is W itself.

Problem 2 (Girth of buildings of type I_2)**.** Recall that the **girth** of a graph Γ is the smallest $k \geq 3$ such that Γ has a k-cycle (∞ if Γ is a forest). Show that if $m \geq 2$, then every building of type $\mathrm{I}_2(m)$ has girth $2m$.

Problem 3 (Local finiteness)**.** Show that a spherical building is finite if and only if each panel is a face of only finitely many chambers.

Problem 4 (A criterion for sphericity)**.**

(a) Let (W, S) be a Coxeter system and suppose $w \in W$ is an element satisfying $D_R(w) = S$; in other words, w is maximal for the right weak order. Show that then (W, S) is spherical and w is the top element w_0 for the Bruhat order.

Hint: Use the lifting property from Sheet 8 to show that $u \leq w$ for every $u \in W$; argue by induction on $l(u)$.

(b) Suppose that two chambers C, D in a building Δ are such that every chamber D' adjacent to D satisfies $d(C, D') \leq d(C, D)$. Show that Δ is spherical and C, D are opposite.