Conference
Structures on Singular Spaces
Through the Lens of Characteristic Classes
September 16 – 20, 2024

Venue: Heidelberg University, Mathematikon,
         Im Neuenheimer Feld 205
         Conference Room, 5th floor

         Registration: Monday, 08:30 – 09.00 am, Lobby

Schedule:

	MON	TUE	WED	THU	FRI
09:00-10:00	Maxim Intro	Banagl Intro	Albin	Tibar	Libgober
10:00-10:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30-11.30	Banagl Intro	Maxim Intro	Piazza	Woolf	Aluffi
13:30-14.30	Yokura	Shaneson		Chataur	Mihalcea
14:45-15:45	de Bobadilla	Budur	afternoon	Waas	Jayasinghe
15:45-16:30	Coffee Break	Coffee Break	at free	Coffee Break	Coffee Break
16:30-17:30	Ludwig	Weber	disposal	Cappell	N.N.

September 16, 2024, 09:00 am   **Laurentiu Maxim**
September 17, 2024, 10:30   **University of Wisconsin-**
                                      Madison

Minicourse: Singularities through the lens of characteristic classes

Abstract: I will start by introducing characteristic numbers and characteristic classes for singular complex algebraic varieties, with an emphasis on Hirzebruch classes. I will then exploit the Hodge theoretic and motivic nature of the Hirzebruch classes to compute them in several

instances, e.g., in the toric context. Several applications, mainly based on joint work with J. Schürmann, will also be discussed.

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September 16, 2024, 10:30　　　　**Markus Banagl**
September 17, 2024, 09:00　　　　**University of Heidelberg**

Minicourse: Orientation classes in K- and L-theory of singular spaces

Abstract: This minicourse will provide an introduction to orientation classes of pseudomanifolds. We will in particular consider the Siegel-Sullivan orientation in KO-homology (away from 2) and its relation both to orientations in Ranicki's symmetric L-homology, and to analytic K-homology classes introduced by Albin, Leichtnam, Mazzeo and Piazza. All of these classes are lifts of the Goresky-MacPherson L-class. Gysin transfer results for these classes, such as VRR-type formulae for the L-class, yield applications to a conjecture of Brasselet, Schürmann and Yokura. Another application is the construction of an equivariant L-class generalizing the Goresky-MacPherson class.

September 16, 2024, 13:30　　　　**Shoji Yokura**
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Title:　　　Characteristic classes and bivariant theories

Abstract: Characteristic classes are natural transformations of contravariant functors or covariant functors. Bivariant theories unify covariant and contravariant functors. In this talk I will give a quick review of them and some related topics and if time permits I will talk about some recent results and a possible application.

September 16, 2024, 14:45　　　　**Javier de Bobadilla**
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Title:　　　Symplectic geometry of degenerations at radius 0
Abstract: Given a normal crossings degeneration $f:(X,w_X) \to D$ of compact Kähler manifolds, in recent work with T. Pelka we have shown how to associate a smooth locally trivial fibration $f_A:X_A \dashrightarrow D\_log$ over

the real oriented blow up of the disc Δ. It is moreover endowed with a closed 2-form w_A giving it the structure of a symplectic fibration. The restriction of w_A to every fibre of f_A «at positive radius» (that is over a point of D\{0} is the modification by a potential of the restriction of w_X to the same fibre. The construction can be regarded as a symplectic realization of A'Campo model for the monodromy. I will highlight the main aspects of the construction, and explain how to apply it to produce Lagrangian torus fibrations over a the complement of a codimension 2 set over the (expanded) essential skeleton of a maximal Calabi-Yau degeneration.

September 16, 2024, 16:30   **Ursula Ludwig**
                                                          Universität Münster

Title: A Morse theoretical complex computing intersection homology

Abstract:
In this talk we present for a compact pseudomanifold and a given perversity in the sense of Goresky and MacPherson a Morse theoretical cochain complex, which computes the intersection cohomology of the space. The complex generalises the famous Morse-Thom-Smale complex associated to a smooth Morse function on a smooth compact manifold to pseudomanifolds equipped with socalled radial Morse functions.

September 17, 2024, 13:30   **Julius Shaneson**
                                                          University of Pennsylvania

Title: Topological String Junctions and Elliptic Fibrations over
                        Curves and Surfaces

Abstract: An elliptic fibration is an algebraic map π: X→B with general fiber a smooth elliptic curve, i.e. a torus (projective case) or a punctured torus (affine case). It will be assumed there is a section. If B is a curve, then the Kodaira classification determines the fibration birationally in the neighborhood of an isolated singularity x of π , i.e. $\pi^{-1}(x)$ is not a

smooth elliptic curve, in terms of an associated Lie algebra of type ADE, with one ambiguity. The Lie algebra arises from an intersection form on the smooth model. We first present an alternative way to arrive at the Lie algebra via the study of the topological string junctions of a deformation, a Morsification. This also gives somewhat more information and in particular removes the ambiguity. In Physics, these fibrations are related to superconformal quantum field theories, the Lie group is the associated gauge group. We will then use the topological string junction method to study a neighborhood of a codimension two singular point for an elliptic fibration over a two-dimensional base. The singular set will have curves and points, in the interior of a curve we can reduce to the previous case, and we will study what happens in the neighborhood of a point to associate gauge groups. The advantage of the string junction method is that smoothness, not always present in this dimension, is not required. In Physics, these structures correspond to the creation of matter. (The next dimension would be couplings in Physics.)
(Joint with A. Grassi, J. Halverson, F. Ruehle, and B. Tian)

September 17, 2024, 14:45　　　**Nero Budur**
Leuven University

Title:　　The local structure of the generic theta divisor

Abstract: A principle governing deformation theory with cohomology constraints in characteristic zero, generalizing Deligne's well known deformation theory principle, was developed together with B. Wang in terms of differential graded Lie modules, and with M. Rubio in terms of L-infinity modules. This principle has been illustrated many situations with complicated moduli spaces, but surprisingly, it has also recently led to a result about one of the oldest topics in algebraic geometry: for a generic compact Riemann surface the theta function is at every point on the Jacobian equal to its first Taylor term, up to a holomorphic change of local coordinates and multiplication by a local holomorphic unit.

September 17, 2024, 16:30　　　**Andrzej Weber**
University of Warsaw

Title:　　Localizing characteristic classes

Abstract: Many interesting singular algebraic varieties admit algebraic group action. Computing their global invariants can be reduced to an analysis of neighbourhoods of the fixed points. We will review localisation techniques for the torus action and show application for computation of characteristic classes. Starting from the fundamental class we will arrive at the elliptic characteristic classes. The main examples are the various versions of Schubert varieties.

September 18, 2024, 9:00   **Pierre Albin**
University of Illinois
Urbana-Champaign

Title: Analytic transfer in K-homology for stratified spaces

Abstract: The classical umkehr map of Hopf assigns to a map of oriented manifolds, f:M -> N, `wrong-way' homomorphisms in homology f_!: H_* (N) -> H_* (M) and in cohomology f^!: H^* (M) -> H^* (N), the latter a version of `integration over the fibers'. Similar wrong-way maps, sometimes known as transfer maps or Gysin maps, are defined for other generalized (co)homology theories as long as the manifolds are suitably oriented and have had many applications. While these maps are defined only for manifolds there has long been interest in extending them to singular spaces. I'll discuss joint work with Markus Banagl and Paolo Piazza in which we capitalize on recent work on the index theory of signature operators to give analytic definitions of transfer maps in K-homology for stratified spaces and relate them to topological orientations.

September 18, 2024, 10:30   **Paolo Piazza**
Sapienza Università di Roma

Title: Topology and Analysis of the equivariant signature operator
on G-Witt spaces

Abstract: Let X be a compact G-Witt pseudomanifold with G a finite group. This means that G acts on X by stratified diffeomorphisms. In this talk I will explain how to use the equivariant signature operator and KK-theory in order to prove an extension of the Atiyah-Singer G-signature formula to G-Witt pseudomanifolds. In the second part of the talk I plan to illustrate how to define an analytic equivariant L-class (which is

expected to agree with the topological equivariant L-class of Banagl up to powers of 2). This is joint work with Markus Banagl and Eric Leichtnam.

September 19, 2024, 09:00   **Mihai-Marius Tibar**

                                                  Université de Lille

Title: Enumerative Geometry of the Gradient

Abstract: I will discuss some recent results involving the degree of the gradient: the polar degree, the Euclidean distance degree (ED-degree), and the linear Morsification of complex polynomials. Some of these degrees enter in the computations of characteristic classes.

September 19, 2024, 10:30   **Jonathan Woolf**

                                                  University of Liverpool

Title: Witt groups and tilted-dual hearts

Abstract: Poincaré duality is a rich source of invariants for manifolds, and is at the core of their classification. Its local origin is the self-duality of the constant sheaf on a manifold. This makes it natural to ask when the constant sheaf on the regular part of a singular space admits a self-dual extension, and more generally to attempt to classify such extensions. The most famous examples are Goresky--MacPherson's intersection cohomology complexes. In general, the answers depend delicately both on the singularities of the space and the coefficients.

The Balmer-Witt groups of the constructible derived category provide a convenient formal framework for studying these questions, and allow one to treat obstructions arising from the singularities and from the coefficients separately. However, this approach is only effective if one can actually compute these groups. The main goal of this talk will be to explain how to reduce this question to computations involving local systems on strata. The key technical tool is the fact that certain categories of perverse sheaves are 'tilted-dual hearts' in the constructible derived category. This is joint work with Jörg Schürmann.

September 19, 2024, 13:30   David Chataur
                                       Université de Picardie Jules
                                       Vernes

Title: Intersection homotopy theory

Abstract: In this talk, I will survey some works on homotopical foundations for Intersection Cohomology. I plan to give some (partial) answers to the following questions :
- What are Intersection homotopy groups?
- What is a generalized Intersection Cohomology Theory?
- What could be the intersection homotopy type of a pseudomanifold?

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September 19, 2024, 14:45   Lukas Waas
                                       Universität Heidelberg

Title: Presenting the stratified homotopy hypothesis

Abstract: Roughly speaking, the homotopy hypothesis - due to Grothendieck - states that the homotopy theory of spaces should be the same as the homotopy theory of infinity-groupoids, where this equivalence is realized by associating to a stratified space its fundamental infinity-groupoid. In the stratified world, the role of the fundamental (infinity)-groupoid is taken by the so-called infinity-category of exit paths. Having this in mind, Ayala, Francis and Rosenblyum conjectured a stratified topological analogue of this principle:
The infinity-category of exit paths should induce an equivalence between the homotopy theory of (topological) stratified spaces and the homotopy theory of layered infinity-categories, i.e. such infinity-categories in which every endomorphism is an isomorphism. We are going to present a formal interpretation of this conceptual conjecture. Namely, we prove the existence of a simplicial semi-model structure for stratified spaces in which most geometrically relevant examples – such as Whitney stratified spaces and PL pseudo-manifolds - are bifibrant. We then prove a Quillen equivalence (in terms of Lurie's exit-path construction) of this semi model category with a left Bousfield localization of the Joyal model structure presenting layered infinity-categories.

September 19, 2024, 16:30

Sylvain Cappell
Courant Institute

Title:

Abstract:

September 20, 2024, 09:00

Anatoly Libgober
University of Illinois, Chicago

Title: Title Equivariant genera and LG/CY correspondence

Abstract: I will discuss algebra-geometric and topological properties of the local contributions given by Atiyah Bott localization of characteristic classes representing Hirzebruch's and elliptic genera of complex manifolds with an action on an algebraic group.

September 20, 2024, 10:30

Paolo Aluffi
Florida State University

Title: The Grothendieck class of the moduli space of pointed stable curves of genus 0

Abstract: The variety M0n-bar parametrizes stable genus 0 curves with n marked points. This is a central object in algebraic geometry, as the most studied and best understood moduli space of curves. Explicit constructions of this variety have been known for several decades, and recursion formulas for its Poincaré polynomial were obtained more than 30 years ago, but (to our knowledge) a more explicit expression for its Betti numbers was not available. We obtain just such an expression, in the form of an explicit generating function for the class of M0n-bar in the Grothendieck group of varieties, and gather more information about related generating functions. As an application, we prove an asymptotic form of log concavity for the Poincaré polynomial of M0n-bar.

September 20, 2024, 13:30   **Leonardo Mihalcea**
                                                        Virginia Polytechnic Institute

Title: Combinatorial applications of characteristic classes

Abstract: I will discuss how invariants associated to characteristic classes of Schubert and Richardson varieties lead to geometric interpretations, or to new proofs, of several combinatorial formulae. Based on joint work with P. Aluffi, B. Ion, H. Naruse, J. Schürmann, and C. Su.

September 20, 2024, 14:45   **Gayana Jayasinghe**
                                                        University of Illinois
                                                        Urbana-Champaign

Title: Holomorphic instanton complexes and applications on
                          stratified pseudomanifolds

Abstract: The classical fixed point theorem of Lefschetz for the case of a self map induced by the gradient flow of a Morse function, is generalized by the classical Morse inequalities. The Lefschetz-Riemann-Roch (LRR) theorem and related localization results are of great interest on both smooth and singular spaces, with various applications in both mathematics and physics. Witten explained how to formulate holomorphic Morse inequalities that generalize the LRR theorem for Kähler Hamiltonian actions on smooth spaces, and to construct categorifications of the Morse polynomials, dubbed instanton complexes for their relation to tunneling. I will describe an extension of these results to singular spaces, and some applications.