Let p be a prime number, let F be a finite extension of $\mathbb{Q}_p$, let G be a split reductive algebraic group over F, let K be a hyperspecial maximal compact open subgroup of G (e.g. $GH = \mathrm{GL}_n(F)$ and $K = \mathrm{GL}_n(\mathcal{O}_F)$). Let R be a commutative ring, let V be a representation of K on a free R-module of finite rank. The compactly induced G-representation $\mathrm{ind}_K^G V$ is tautologically a module over the spherical Hecke algebra $\mathfrak{h}_V = \mathrm{End}_{R[G]} v$. We want to explain why – for suitable V over $R = \overline{\mathbb{R}}_p$ or over finite extensions R of $\mathcal{O}_F$ – it is of great interest for the p-modular and the p-adic representation theory of G to know if $\mathrm{ind}_K^G V$ is free as a $\mathfrak{h}_V$-module.
So far this freeness is known only if $G = \mathrm{GL}_2(F)$, by work of Barthel and Livné. We want to explain that their result can be extended to a large class of more general G, at least under certain hypotheses on V. For example, freeness holds true if $G = \mathrm{PGL}_n(\mathbb{Q}_p)$ and V is a rational representation of $\mathrm{PGL}_n(\mathbb{F}_p)$ (and hence a representation of $K = \mathrm{PGL}_n(Z_p)$) with 'p-small´(in a suitable sense) highest weights.