On Mordell-Weil, Tate-Shafarevich and Bloch-Kato

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This is joint work with D. Burns and C. Wuthrich. Given an abelian variety A defined over a number field k, a prime number p and a finite Galois extension F of k, we describe how, under some not-too-stringent conditions, there is a very strong interplay between the $\mathbb{Z}_p[\mathrm{Gal}(F/k)]$-module structures of $\mathbb{Z}_p \otimes_{\mathbb{Z}} A(F)$ and of the p-primary Tate-Shafarevich group of A over F. This allows us in certain cases to prove structure results about $\mathbb{Z}_p \otimes_{\mathbb{Z}} A(F)$, which we will in turn use to make completely explicit the relevant case of the equivariant Tamagawa number conjecture. In particular, we obtain the first (theoretical and numerical) verifications of the p-part of the ETNC in the technically most demanding case in which $A(F)$ has strictly positive rank and $\mathrm{Gal}(F/k)$ is non-abelian and of order divisible by p.